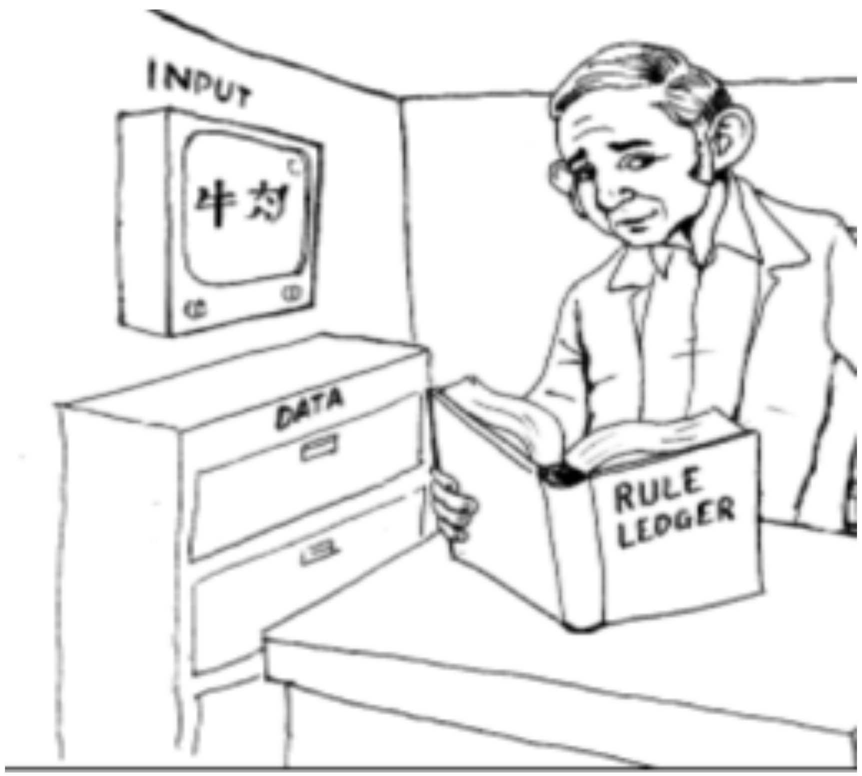
CSE322 Theory of Computation (L11)

Recap



Today

Pushdown Automata

$R = R1 \cup R2$

$L(R) = L$

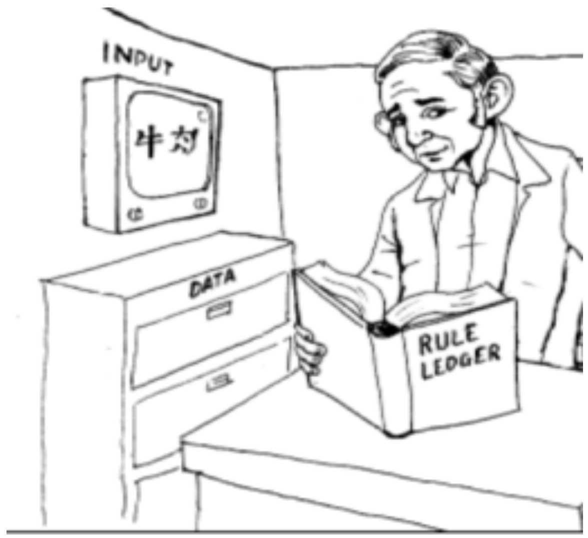
$L(R) \subseteq L \rightarrow$ if x matches R then $x \in L$

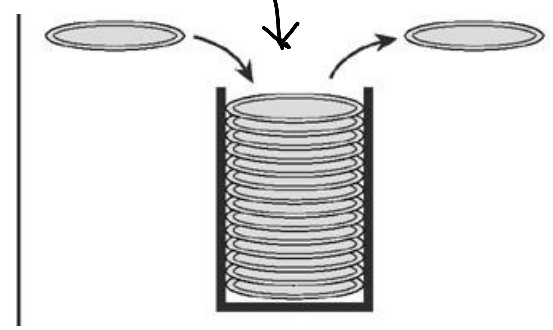
$L \subseteq L(R) \rightarrow$ if $x \in L$ then x matches R

Does not appear good for clustering, scheduling, optimization, numerical ananlysis, program (syntax, semantics) validation, ...

PDA

push down automata





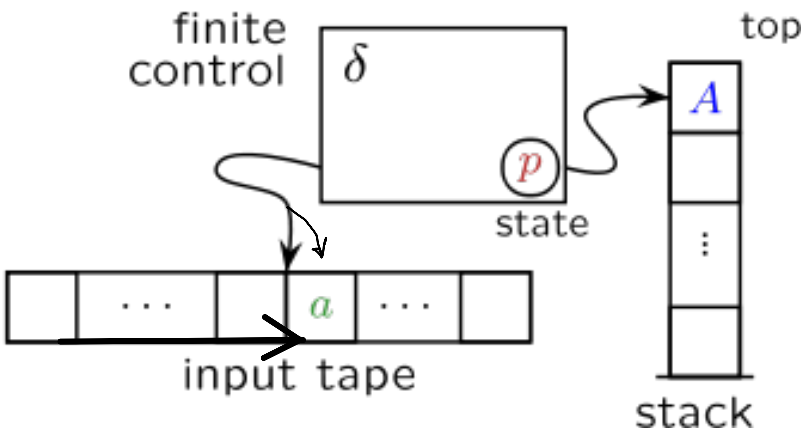
Automata + Data structure

CFL context-free language

$REG \subsetneq CFL$

NPDA vs DPDA

Non-deterministic Pushdown Automata (PDA)



stack usage : pop & push at every step

use ε to ...

not read from input

not pop from stack

not push into stack

$q_7 \xrightarrow{a,b \to c} q_5$

apply this transition when
- next symbol of input is a
- b should be at the top of the stack

q1/q2/q2/q2/q2/q3/q3/q3/q4

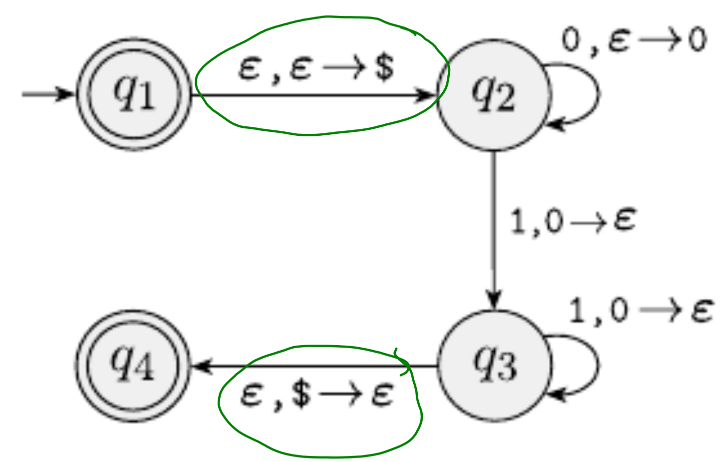
$Q = \{q1, q2, q3, q4\}$

input alph. = $\{0,1\}$

stack alph. = $\{0,\$\}$

$q0 = q1$

$F = \{q1, q4\}$



input: 000111

input: 00011 $\notin L$

input: 00111 $\notin L$

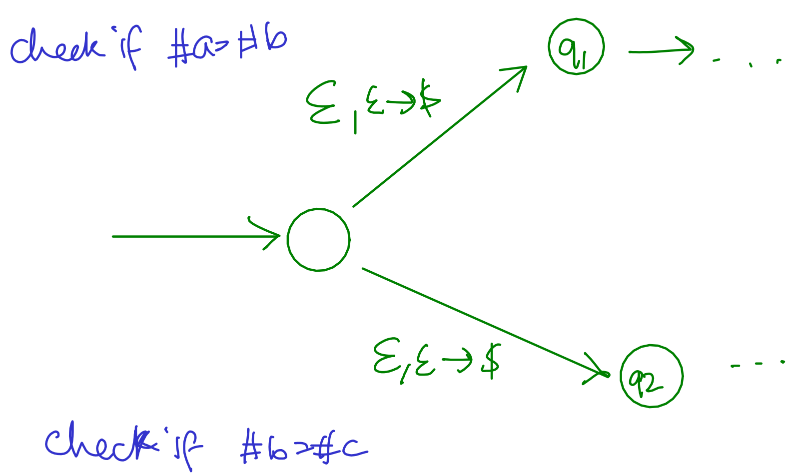
$L = \{ w : w \in \{0,1\}^* \text{ s.t. } w = 0^k 1^k, k \geq 0 \}$

$q2 \xrightarrow[0,\varepsilon \to 0]{} q2$

Input:	0			1			ε		
Stack:	0	\$	ε	0	\$	ε	0	\$	ε
q_1									$\{(q_2,\$)\}$
q_2			$\boxed{\{(q_2,0)\}}$	$\{(q_3,\varepsilon)\}$					
q_3				$\{(q_3,\varepsilon)\}$				$\{(q_4,\varepsilon)\}$	
q_4									

Exercise: $\{a^i b^j c^k : i=j \text{ or } j=k\}$

$a^i b^j c^k$; $i=j$ or $j=k$



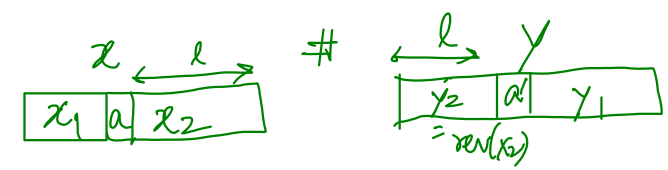
Construct PDA for $L = \{ x\#y : x \mathrel{!=} rev(y) \}$

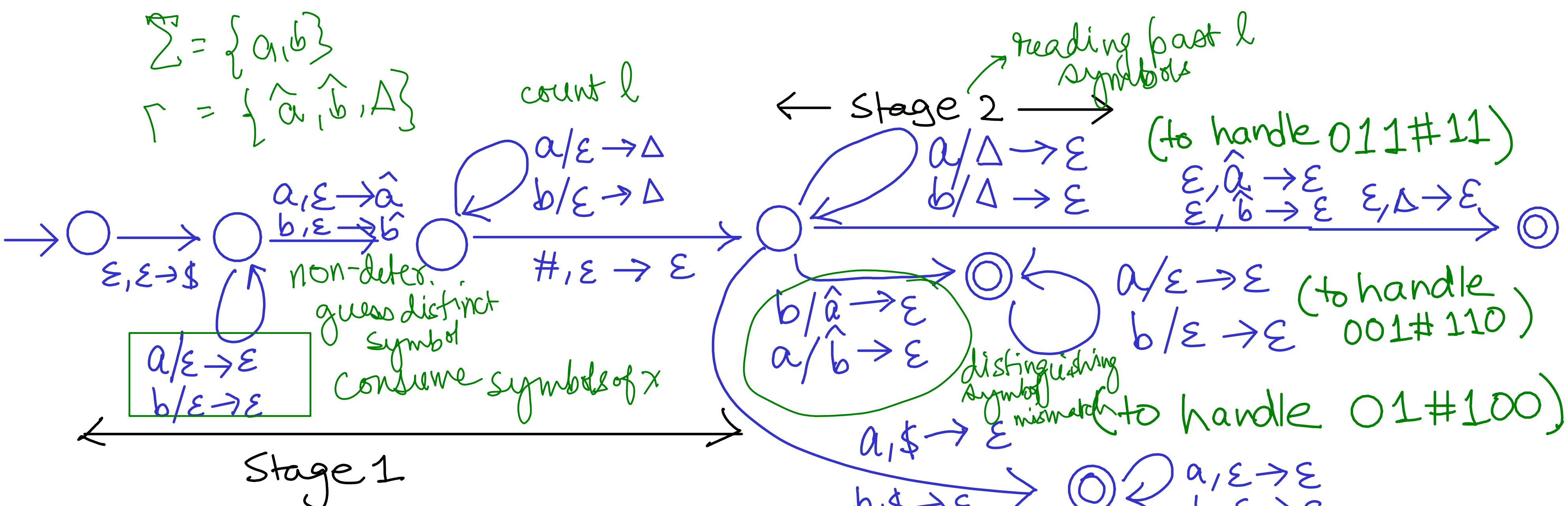
01#111
01#11
01#10 X

Exercise $\{x\#y : x \neq y\}$ $\{x\#y : x = y\}$

1. Stage-1: Keep reading all symbols until # is seen. While reading, non-det. go to 2.
2. Non-deterministically mark one symbol and push it into the stack. Go to 3.
3. Push all remaining symbols into the stack until # is seen.
3. When # is seen move to stage-2 at 4.
4. Stage-2: Keep popping all symbols and reading input symbols until a marked symbol is seen on stack.
5. When marked symbol is seen, check if it matches the current input symbol.
6. If the current input symbol and marked symbol on stack do not match, accept. O/w go to a trap state.

$a \neq a'$





Context-free Grammar

rule: variable → string over variables & terminals

start variable $A \to 0A1$

substitution rules $A \to B$

/ productions $B \to \#$

terminal $= \{0, 1, \#\}$

variable $= \{A, B\}$

$A \to B \to \#$

$A \to 0A1 \to 00A11 \to 000A111$
$\to 000B111$
$\to 000\#111$

Derivation: Steps to generate a string

Context Free Language (CFL) : Strings generated by CF grammar

$L(G) = \{w : G \text{ produces } w\}$

Ex: Language of above grammar?

$L = \{w : w \text{ is of the form } 0^n\#1^n, n \geq 0\}$

CFG

$A \to 0A1$; $A \to B$ $\}$ $A \to 0A1 \mid B$

$A \to 0A1$ $\quad (A, 0A1)$: $A \in V$, $0A1 \in (V \cup \Sigma)^*$ $\Rightarrow$ empty string

A CFG is (V, Σ, R, S)

- V set of variables
- Σ set of terminal symbols
- R : as a tuple
- $S \in V$: starting variable

$$\langle \text{EXPR} \rangle \to \langle \text{EXPR} \rangle + \langle \text{TERM} \rangle \mid \langle \text{TERM} \rangle$$
$$\langle \text{TERM} \rangle \to \langle \text{TERM} \rangle \times \langle \text{FACTOR} \rangle \mid \langle \text{FACTOR} \rangle$$
$$\langle \text{FACTOR} \rangle \to (\langle \text{EXPR} \rangle) \mid a$$

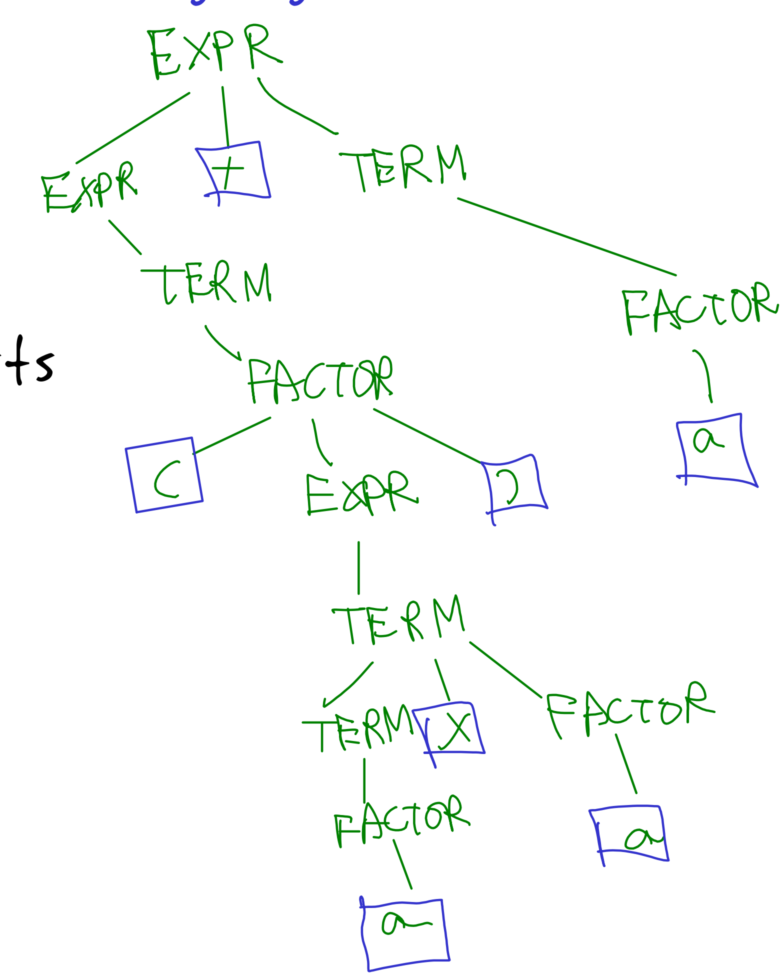
$V = \{\langle \text{EXPR} \rangle, \langle \text{TERM} \rangle, \langle \text{FACTOR} \rangle\}$

$\Sigma = \{+, \times, (,), a\}$

$S = \langle \text{EXPR} \rangle$

Is "(axa)+a" present in language of the above?

Yes



Derivation:

u,v,w: string over variables & terminals

A : variable

- uAv => uwv (uAv yields uwv) if A -> w rule exists
- u =>* v (u derives v) if

  u = v, or

  u => v1 => v2 => ... => v for some v1, v2, ...

Language (G) = { w over terminals | S =>* w}

Ex. Write a grammar to generate balanced string of brackets, braces, ~~etc~~.

balanced

w case: starts and ends w/ $(,) \to w = (\square)$

" " " " $\{,\} \to w = \{\square\}$

$S \to (S) \mid \{S\} \mid \varepsilon$ "( )"

Ex. Write a grammar to generate palindromes.

$L(G) = \{ w : w = w^{rev} \}$ over $\{0, 1\}$ $S \to \varepsilon \mid 0 \mid 1 \mid 0S0 \mid 1S1$

x is in $L(G)$ iff (a) $x = \varepsilon$, or

(b) $x = 0$, or

(c) $x = 1$, or

(d) $x = 0y0$ or $1y1$ where $y = y^{rev}$

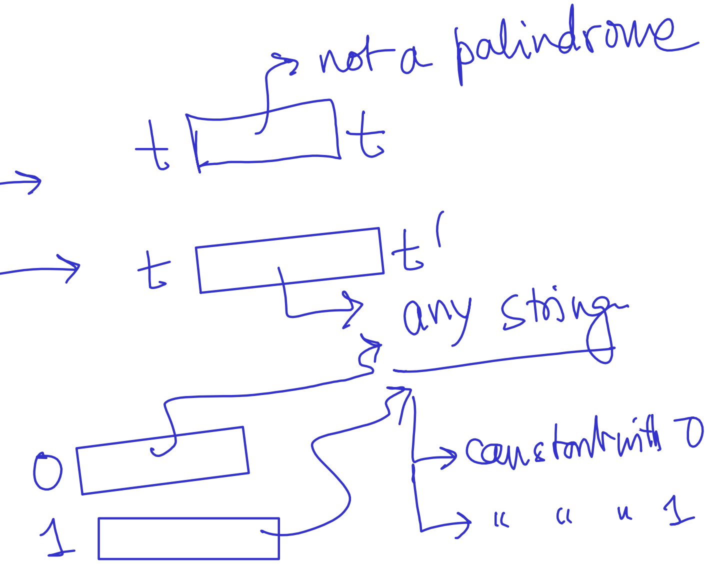
$S \to 011110$

CFG for non-palindrome $\{w : w \neq w^{(rev)}\}$ over $\{0,1\}$

w is not a palindrome. w can be of types:

(1) w starts and ends with the same symbol

(2) w starts and ends with different symbol



smallest string : 01 , 10

$$S \longrightarrow 0S0 \mid 1S1 \mid 0A1 \mid 1A0$$

$$A \text{ (any string)} \longrightarrow 0A \mid 1A \mid \varepsilon$$